Quadratic vector fields in Class I

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Abstract

Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers have been written on these systems, a complete understanding of this class is still missing. Classical problems, and in particular, Hilbert's 16th problem [28, 29], are still open for this class. Among many interesting families of quadratic systems, the ones which may have limit cycles are of specially interest. In [42], these systems where classified in three different normal forms (I, II and III) with increasing number of parameters. The simplest family is I and even several subfamilies of it have been studied, and some global attempts have been done, up to this paper, the full study was still undone.

In this article we make an interdisciplinary global study of the class I. Since the family has 4 parameters, we have studied it using the same technique that has already been used in several papers with similar systems which is based in the algebraic invariants of the Sibirskii's school.

The bifurcation diagram for this class, done in the adequate parameter space which is the 3–dimensional real projective space, is quite rich in its complexity and yields 261 subsets with 49 different phase portraits for Class I (two of them corresponding to linear systems), seven of which have limit cycles. The phase portraits are always represented in the Poincaré disc. The bifurcation set is formed by an algebraic set of bifurcations of singularities, finite or infinite and by an analytic set of curves corresponding to phase portraits which have separatrix connections.

Algebraic invariants were needed to construct the algebraic part of the bifurcation set, symbolic computations to deal with some quite complex invariants and numerical calculations to determine the position of the analytic bifurcation set of connections.

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